Exercise 6

Find the area of a triangle bounded by the x-axis, the line $f(x) = 12 - \frac{1}{3}x$, and the line perpendicular to $f(x)$ that passes through the origin.

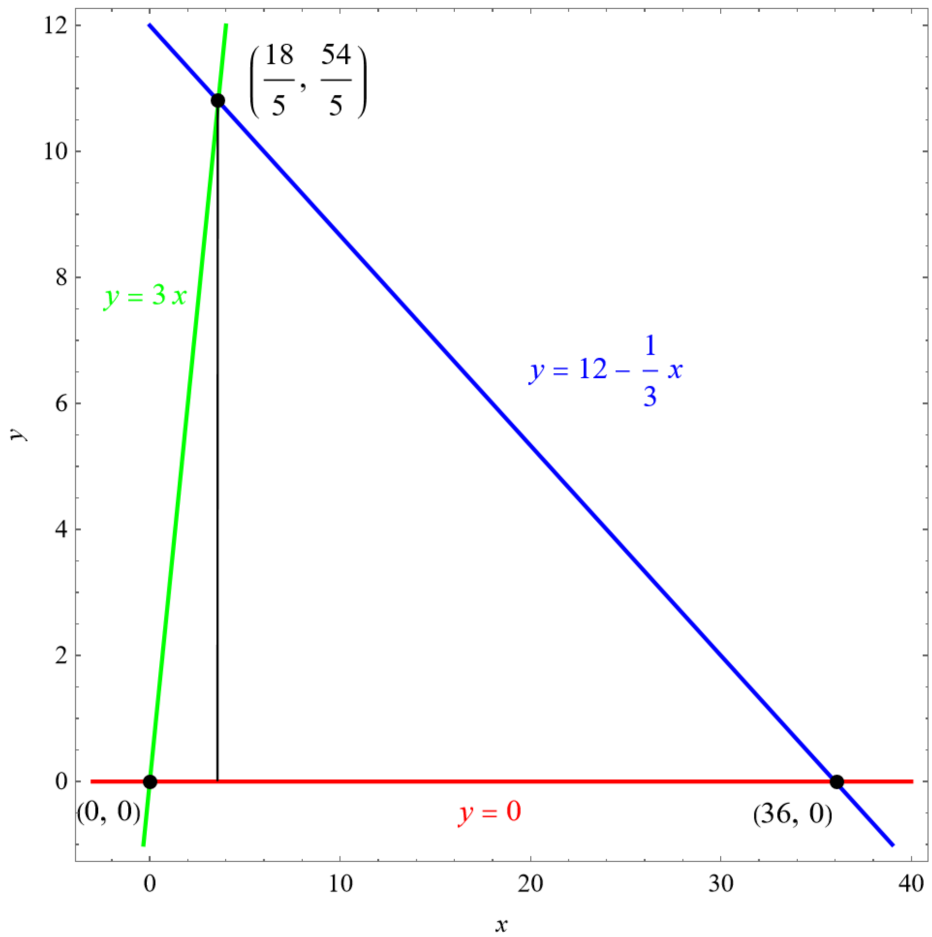
Solution

Start by writing equations of the lines that are given. The equation for the x-axis is $y = 0$, $y = 12 - \frac{1}{3}x$ is given, and the line perpendicular to $f(x)$ has the negative reciprocal slope (3) with an equation given by the point-slope formula.

$$y - 0 = 3(x - 0)$$

$$y = 3x$$

Now graph all of them.



The area of the triangle is half the product of the base and height.

$$A = \frac{1}{2}bh = \frac{1}{2}(36)\left(\frac{54}{5}\right) = \frac{972}{5}.$$

The point of intersection at the top is found by setting the top two functions of x equal to each other and solving for x.

$$3x = 12 - \frac{1}{3}x$$

$$3x + \frac{1}{3}x = 12$$

$$\frac{10}{3}x = 12$$

$$x = \frac{36}{10} = \frac{18}{5}$$

Plug this value of x into either of the two functions to determine the corresponding y-value.

$$y = 3\left(\frac{18}{5}\right) = \frac{54}{5}$$

This means the intersection point at the top is $\left(\frac{18}{5}, \frac{54}{5}\right)$. The point of intersection at the bottom right is found similarly by setting the bottom two functions of x equal and solving for x.

$$12 - \frac{1}{3}x = 0$$

$$-\frac{1}{3}x = -12$$

$$x = 36$$

Since $y = 0$ at this point, the bottom point of intersection is $(36, 0)$.